International Journal of Mechanical and Production Engineering Research and Development (IJMPERD)
ISSN (P): 2249-6890; ISSN (E): 2249-8001
Vol. 8 Issue 5, Oct 2018, 359-374

CONTACT STRESS ANALYSIS & OPTIMIZATION OF SPUR GEAR BY FINITE ELEMENT ANALYSIS

SUHAS BORATE & PRASAD P. KULKARNI

Department of Mechanical Engineering, SKN Sinhgad College of Engineering, Pandharpur, Maharashtra, India

ABSTRACT

The gears are the most common means to transmit the power. Spur Gears are used in many machine tools to transmit the power like Lathe machines, Drilling Machines etc. Since the Spur gears are subjected to high loads in such machine tools, results in high contact stresses between mating gear tooth. As a result of that, gear tooth failures occur, so it is necessary to reduce it. The purpose of this dissertation work is to identify the magnitude of the stresses for a Specified design and to optimize the weight of spur gear. Analysis and Optimization of spur gear is done by using Finite Element tool. Results are compared with Hertz contact stress. By changing the current conventional design parameters we can achieve better performance of the gear under contact pressure as well as in equivalent stress while reducing the weight of the gear. It can be clearly observed that 30.01 % of Equivalent stress, 10.21% of contact pressure is reduced& 12.51% of total mass is reduced when compared to baseline gear assembly with modified design.

KEYWORDS: Spur Gear, Contact Stress and Fatigue Life & Finite Element Analysis

Received: Jul 26, 2018; **Accepted:** Aug 16, 2018; **Published:** Sep 14, 2018; **Paper Id.:** IJMPERDOCT201841

Original Article

1. INTRODUCTION

Gears are defined as toothed wheels or multi lobed cams, which transmit power and motion from one shaft to another by means of successive engagement of teeth. In case of Spur gears, the teeth are cut parallel to the axis of the shaft; spur gear is used only when the shafts are parallel. The profile of the gear tooth is in the shape of an involute curve and it remains identical along the entire width of the gear wheel. Spur gears impose radial loads on the shafts. Spur gears are the most common type of gears. They have straight teeth, and are mounted on parallel shafts. Sometimes, many spur gears are used at once to create very large gear reductions.

Spur gears are used in many devices like the electric screwdriver, dancing monster, oscillating sprinkler, windup alarm clock, washing machine and clothes dryer.

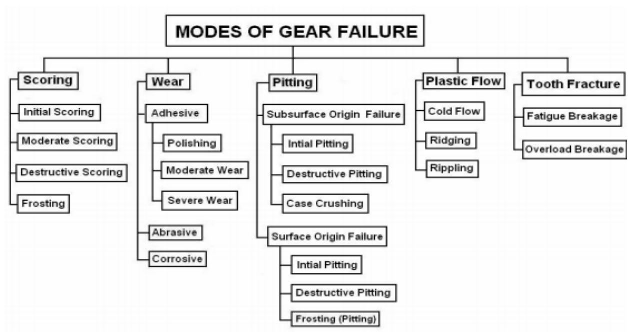

Figure 1: Flow Diagram for Modes of Gear Failure

It is assumed that, as this small area of contact forms, points that come into contact are points on the two surfaces that originally were equal distances from the tangent plane. There are two types of pitting-Initial and destructive Pitting. The initial pitting is a localized phenomenon, characterized by small pits at high spots. Initial pitting is caused by the errors in the tooth profile, surface irregularities and misalignment. The Destructive pitting is a surface failure, which occurs when the load on the gear tooth exceeds the surface endurance strength of the material. Destructive pitting depends upon the magnitude of the Hertz's contact stress and number of stress cycles. This type of failure is characterized by pits, which continue to grow resulting in complete destruction of the tooth surface. It is common practice to design the gear teeth on the basis of calculations and excessive tests which are used to validate calculations. With advancement of technology now we can go for third method of verification of the design by the means of FEA analysis. Many techniques are available to simulate the contact stresses but correct one should be selected and methodology should be finalized to reduce the contact stresses in the conventionally designed gear pairs and advancements in the design on the basis of the same is needed.

2. OBJECTIVES & METHODOLOGY

- To measure and model the current gear pair of lathe gears and calculate their contact stresses mathematically.
- To perform FEA analysis on the modeled gear and finding contact stresses using FEA.
- Optimization of the gear pair through parametric study for reduction of contact stresses while reducing the weight of the geometry.
- Manufacturing of the changed design of gear pair and practical testing to validate the FEA results.

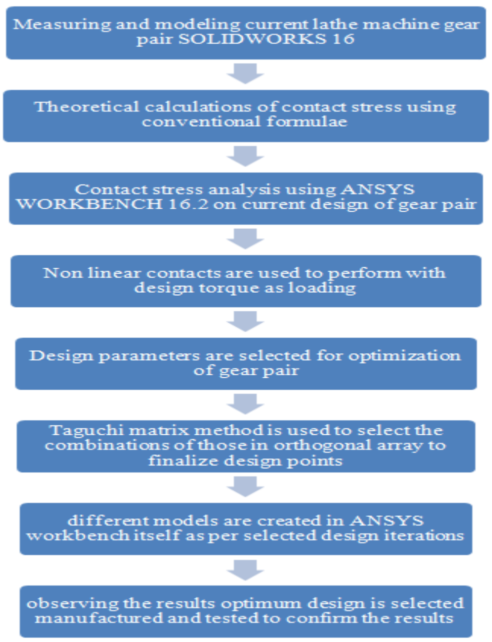

Figure 2: Methodology of the Work

There has been a great deal of research on gear analysis, and a large body of literature on gear modeling has been published. The contact stress analysis of spur gear is done by many researchers but reduction in contact stresses along with mass reduction is not studied yet. Various research findings are described below on the field of spur gear and their terminology. **Dhavale A. S et.al. [1]**Study deals with minimizing the stress in root fillet region of the gear tooth.

Inserting stress relief features (holes) of different diameter at different locations (Beside root fillet) & varying number of holes. A maximum of 15% reduction in maximum principal stress is obtained. As the Number of holes & hole diameter increases the maximum principal stress decreases but after a certain extent the strength of the gear reduces. **Ali Raad Hassan[2]** Study deals with finding of Contact stresses and Contact ratio between two spur gear teeth for different angular positions. He considered steel (C45) material for the gears. The pair of mating gear is rotated through 3^0 angular position and contact stress and contact ratio values are calculated. The maximum stress result obtained from AGMA stress calculation method and the maximum contact stress obtained from the finite element contact analysis are approximately same. The results shows a high value of contact stress in the beginning of the contact, and then it starts to reduce until it reaches the location of single tooth contact, here it increased to the maximum value of the contact, but exactly after exceeding this single contact region it was reduced. At the end of the contact, the stress increased suddenly to a high value almost close to the maximum value, at this stage a sliding was occurred in the contact region at the maximum stress points. **Vivekkaraveer et.al.[3]** study deals with stress analysis of mating teeth of spur gear to find maximum contact stress in the gear teeth. In this, Two gears are used-Steel and Grey Cast Iron for the study. Contact Stresses are calculated for both the gears theoretically by using Hertz's equation and Finite Element Analysis. The contact stresses calculated are well within the Yield Strength of the gear materials. The Values of calculated contact stresses for both the gears by theoretical and analytical method are comparable. Also the deformations under load for both the gears are calculated.

3. THEORETICAL CALCULATION OF CONTACT STRESSES BY ANALYTICAL METHOD (HERTZ EQUATION)

In this, grey cast iron is used as the spur gear materials. The material properties of grey cast iron are given in Table 1.

Table 1: Material Property of Grey Cast Iron

Material Property	Symbol	Value	Unit
Density	P	7100	Kg/m^3
Poisson Ratio	Θ	0.26	-
Young's Modulus	E	1.65E+05	MPa
Tensile Yield	S_{yt}	250	MPa
Tensile Ultimate	S_{ut}	350	MPa

Table 2: Dimensions of Spur Gear

Dimension	Symbol	Gear	Pinion	Unit
No. of Teeth	Z	100	56	-
Pitch Circle Dia	D	160	87.8	mm
Pressure Angle	Φ	14.5	14.5	Degree(^{0})
Addendum Radius	R_A	80.95	46.2	mm
Dedendum Radius	R_D	77.5	42.5	mm
Face Width	B	15.3	16.2	mm
Shaft Radius	R_S	22.2	22.2	mm
Module	M	1.6	1.57	mm

In this study, maximum contact stress is determined, during the transmission of torque of about 6000 N-mm for grey cast iron spur gears, using finite element analysis.

Hertz's equation is given by,

$$\sigma_c = \frac{\sqrt{F(1+\frac{R1}{R2})}}{\sqrt{R1B\pi[\frac{1-\vartheta 1^{\wedge}2}{E1}+\frac{1-\vartheta 2^{\wedge}2}{E2}]Sin\phi}}$$

Where, σ_c is the contact stress in mating teeth of spur gear, F is the force, and R_1, R_2 are pitch radii of two mating gears, B is the face width of gears, ϕ is the pressure angle, ϑ_1,$\vartheta_2$are the Poisson ratios and E_1, $E_2$are the module of elasticity of two gears in mesh.

Allowable maximum stress is given by,

$$\sigma c = \frac{\sigma c}{FOS}$$

Here FOS is the factor of safety which can be taken from the ANSYS results or other tables

The relation between Power and RPM is:

$$P = \frac{2\pi NT}{60\times 10^{\wedge}3}$$

$$T = \frac{P\times 60\times 10^{\wedge}3}{2\pi N}$$

Since,

We have,

Power, P= 1 HP= 746 Watt

N = RPM of driver (1200 RPM)

π = 3.14

Therefore,

Torque T = 5936.48 N-mm.

Now, Torque is given by,

$$T = F \times r$$

Force, F=267.40 N

By using Hertz Equation,

$$\sigma c = \frac{\sqrt{F(1+\frac{R1}{R2})}}{\sqrt{R1B\pi[\frac{1-\vartheta 1^{\wedge}2}{E1}+\frac{1-\vartheta 2^{\wedge}2}{E2}]Sin\phi}}$$

$$\sigma c = \frac{\sqrt{267.40(1+\frac{80}{43.9})}}{\sqrt{80*15.3*\pi[\frac{1-0.26^{\wedge}2}{165*10^{\wedge}3}+\frac{1-0.26^{\wedge}2}{165*10^{\wedge}3}]Sin(14.5)}}$$

$$\boldsymbol{\sigma c = 237.67MPa}$$

Allowable maximum stress is given by,

$$\sigma a = \frac{\sigma c}{FOS}$$

Here, Factor of Safety (FOS) is selected as 1 for Grey Cast Iron

$$\sigma a = \frac{237.67}{1}$$

σa = 237.67 MPa

4. SPUR GEAR STRUCTURAL OPTIMIZATION RESULTS

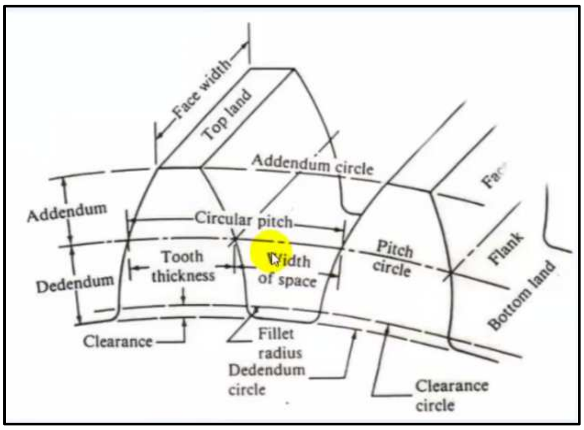

Figure 3: Nomenclature of the Gear Tooth Profile

Design Procedure

Pitch circle radius = (Module × Number of teeth's)/2

Addendum radius = Radius of pitch circle + Module

Deddendum radius = Radius of pitch circle – 1.25× Module

Clearance radius = Deddendum radius + 1.25× Module

PINION

Module (m) = 1.6mm

Number of teeth's (N) = 56

Addendum circle Radius (Ra) = 46.4 mm

Dedendum circle Radius (Rd) = 42.8 mm

Pitch circle Radius (Rp) = 44.8 mm

Clearance circle Radius (Rb) = 43.2 mm

Involute angle = 1.6 degree

Face width = 20 mm

GEAR

Module (m) = 1.6 mm

Number of teeth's (N) = 100

Addendum circle Radius (Ra) = 81.6 mm

Dedendum circle Radius (Rd) = 78 mm

Pitch circle Radius (Rp) = 80 mm

Clearance circle Radius (Rb) = 78.4 mm

Involute angle = 0.9 degree

Face width = 20 mm

Table 3: Baseline Design Parameters

Gear & Pinion Thickness (mm)	Gear & Pinion Fillet Radius (mm)
4	0.6

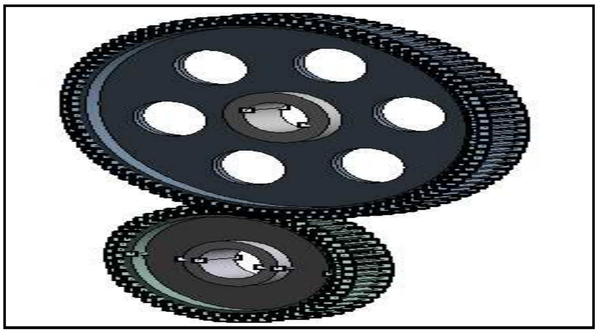

Figure 4: Baseline Geometry of Model

Bonded type contact is defined between the faces of gear & pinion.

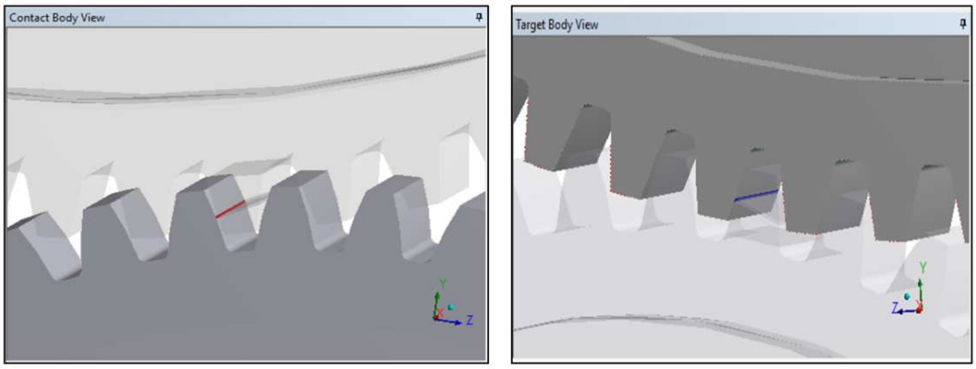

Figure 5: Contact of Gears Baseline

The assembly is meshed with tetrahedron higher order elements with 1207498 nodes &492052 elements.

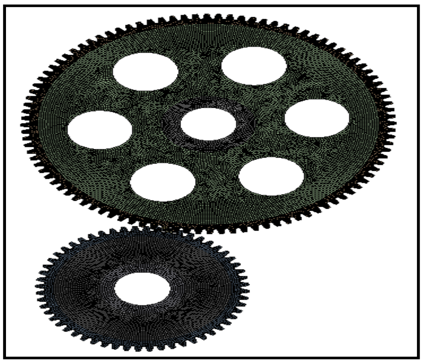

Figure 6: Meshing of Gears Baseline

The shaft mounting region of the gear is fixed & shaft mounting region of the pinion is defined with frictionless support which is free to rotate. The moment of 5936.5 N-mm is applied at the shaft mounting region of pinion in clockwise direction w.r.t x axis as shown in the figure below.

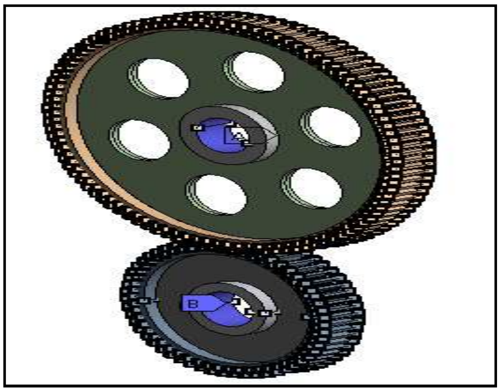

Figure 7: Boundary Conditions Baseline

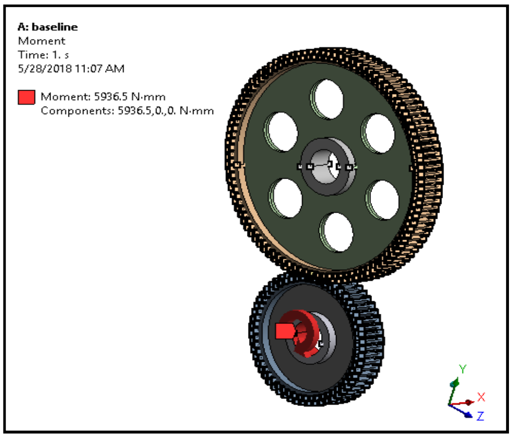

Figure 8: Loading Conditions Baseline

The maximum equivalent stress is found to be 32.289MPa at the contact region & minimum at other than contact region

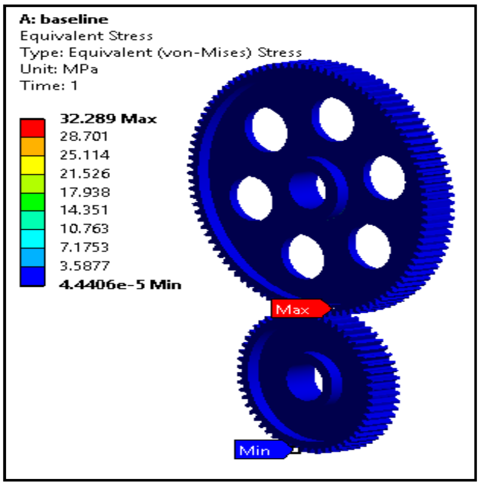

Figure 9: Equivalent Stress Baseline

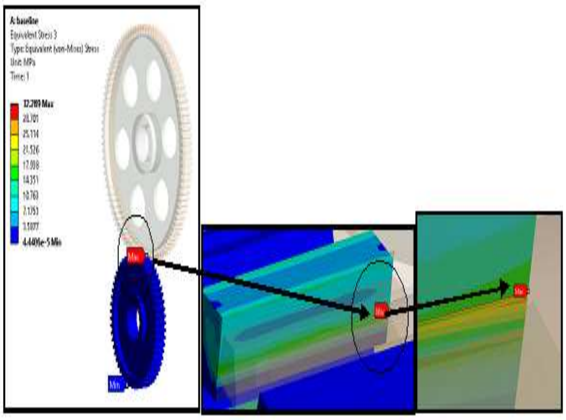

Figure 10: Equivalent Stress Details Baseline

The Maximum total deformation is found to be 0.0028517 mm at the tooth region of the pinion & minimum at the fixed region of the gear.

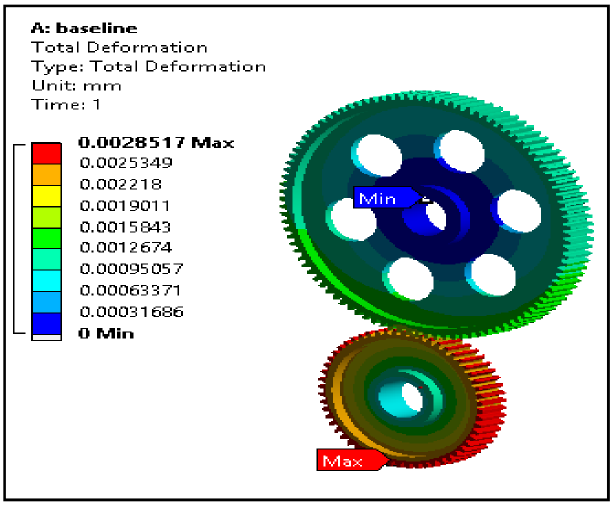

Figure 11: Total Deformation Baseline

The unaveraged maximum pressure at the contact region is found to be 238.38 MPa.

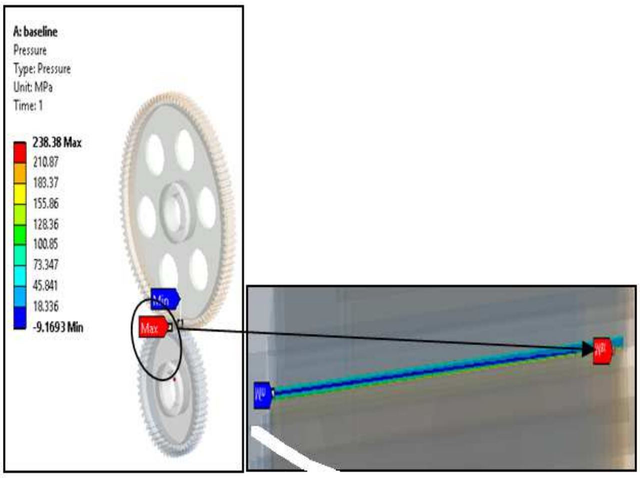

Figure 12: Contact Pressure Baseline

Table 4: Design Parameters for All 9 Iterations

Iteration No.	Hole Diameter (mm)	No. of Holes on Gear & Pinion	Thickness (mm)	Fillet Radius (mm)
1	12	6	4	0.45
2	12	3	3	0.6
3	12	2	2	0.75
4	10	6	3	0.75
5	10	3	2	0.45
6	10	2	4	0.6
7	8	6	2	0.6
8	8	3	4	0.75
9	8	2	3	0.75

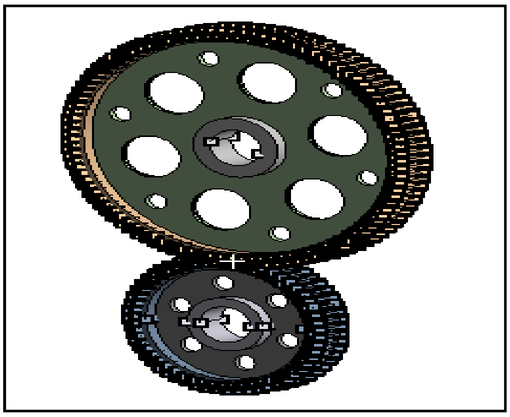

Figure 13: Geometry of Gears Iteration 4

The maximum equivalent stress is found to be 22.596MPa at the contact region & minimum at other than contact region

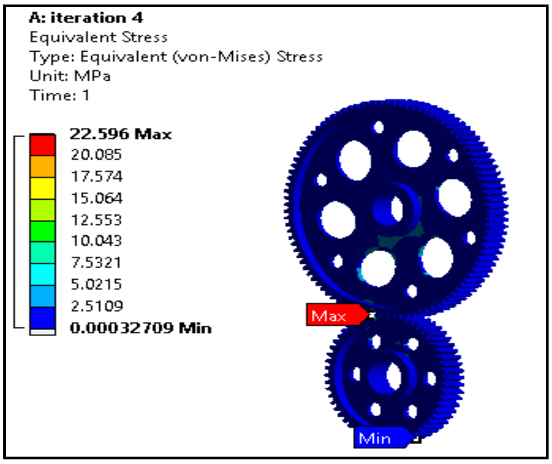

Figure 14: Equivalent Stress Iteration 4

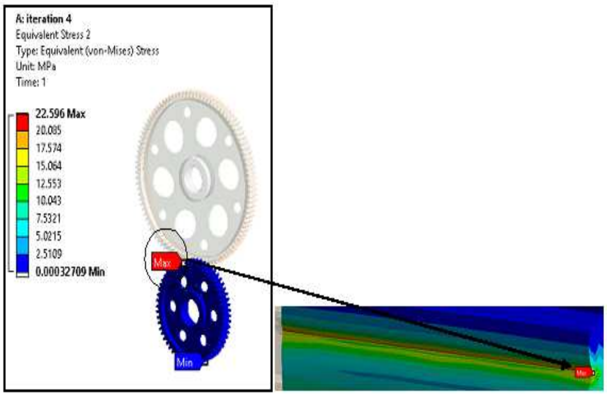

Figure 15: Equivalent Stress Iteration 4

The Maximum total deformation is found to be 0.0033158 mm at the tooth region of the pinion & minimum at the fixed region of the gear.

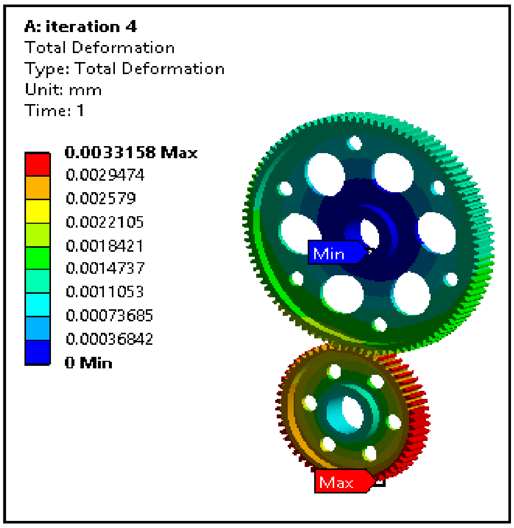

Figure 16: Total Deformation Iteration 4

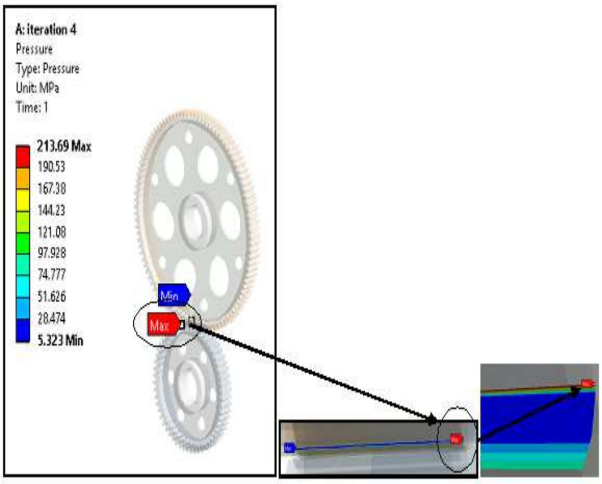

Figure 17: Pressure Iteration 4

All the analysis results are listed in the table below. From those results final design of the gears will be selected

Table 5: Result Summary of all the Iterations in FEA

Iteration	Equivalent Stress (MPa)	Total Deformation (mm)	Pressure (MPa)	Mass of Gear (g)	Mass of Pinion (g)	Total Mass (g)
Baseline	32.239	0.00285	233.33	748.42	346.26	1094.68
1	39.078	0.00302	276.05	726.62	324.67	1051.29
2	33.206	0.0034	246.25	651.41	311.57	962.98
3	24.935	0.00415	236.63	567.53	289.68	857.21
4	22.596	0.00331	213.69	648.92	308.81	957.73
5	27.76	0.00424	308.1	566.28	288.9	855.18
6	32.425	0.00287	238.35	743.48	341.32	1084.8
7	28.817	0.00425	286.06	565.73	288.15	853.88
8	21.64	0.00278	206.81	744.3	341.32	1086.12
9	38.64	0.00332	280.25	656.55	316.91	973.46

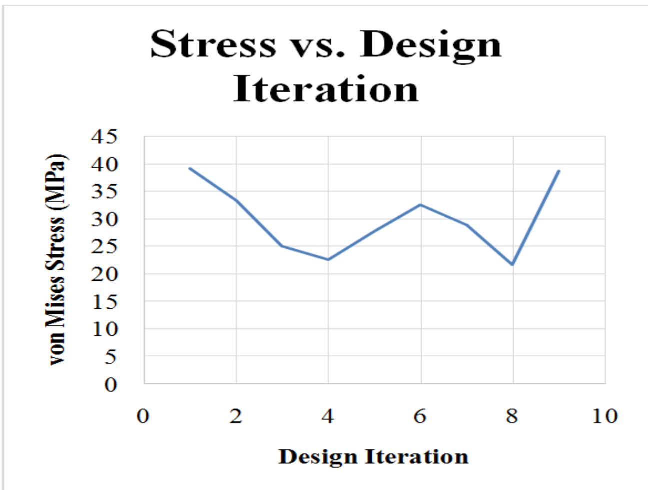

Figure 18: Maximum Von Mises Stress Vs Design Iteration

Iteration wise von-mises stress plot shows that iteration 8 has lowest value of von mises stress 21.6 MPa in all the iterations. While iteration 4 shows the second lowest 22.6 MPa.

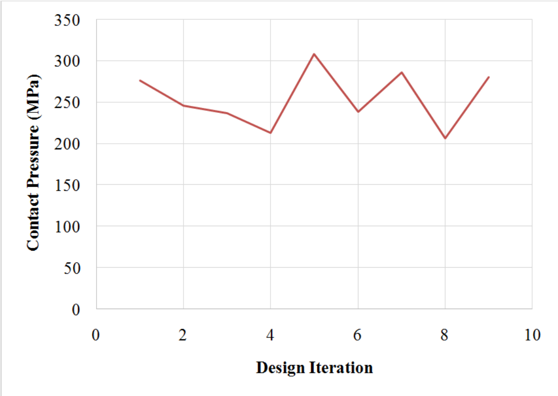

Figure 19: Design Iteration Vs Contact Pressure

Similar pattern to von Mises stress is followed by contact pressure plot it shows the lowest value of 206 MPa while 213 MPa in iteration 4 is observed.

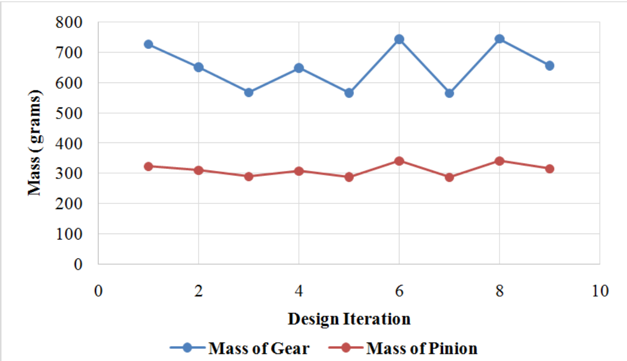

Figure 20: Mass of Gears Vs Design Iterations

Table 6: Comparison of Baseline and Optimized Gears

Gear Assembly	Equivalent Stress (MPa)	Contact Pressure (MPa)	Mass of Gear (gm)	Mass of Pinion (gm)	Total Mass (gm)
Baseline	32.289	238	748.42	346.26	1094.68
Optimized	22.596	213.69	648.92	308.81	957.73
Percentage reduction	30.01%	10.21%	13.29%	10.81%	12.51%

5. TESTING SET UP & RESULTS

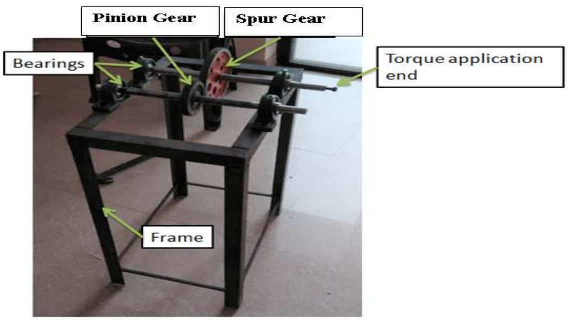

Figure 21: Testing Setup with Iteration 4 Designed Gears

The setup for the gear is designed and manufactured for calculating the torque applied and strain in the contact region of the gear. Figures below give the images at different stage of manufacturing.

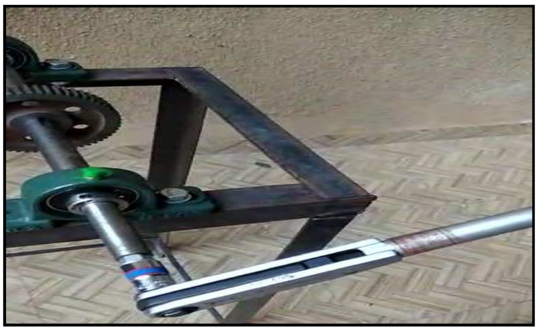

Figure 22: Torque Applied on the Gear Test Set Up

The required torque is selected on the torque wrench and the torque is applied. The strain gauges are attached to the contact region of the mating pair of teeth of the gears and the torque is applied using the torque wrench. One gear is fixed and torque is applied on the other gear. The change in resistance in the strain gauge is observed using the digital multi-meter and the values are recorded. The following table shows the values of the resistance observed during experiment.

Table 7: Resistance Observed in Testing

Initial Resistance in Strain Gauge (ohm)	Final Resistance in Strain Gauge on Application of Torque (ohm)	Change in Resistance
120	120.025	0.025

The basic relationship between strain and the change in gauge resistance can be expressed by:

e = (1 / F) (dR / R)

Where e is the strain,

F is the gauge factor and

R is the gauge resistance.

For a typical gauge F is 2.0 and R is 120 ohm.

$e = \frac{1}{2} \times \frac{0.025}{120}$

$e = 0.000104$

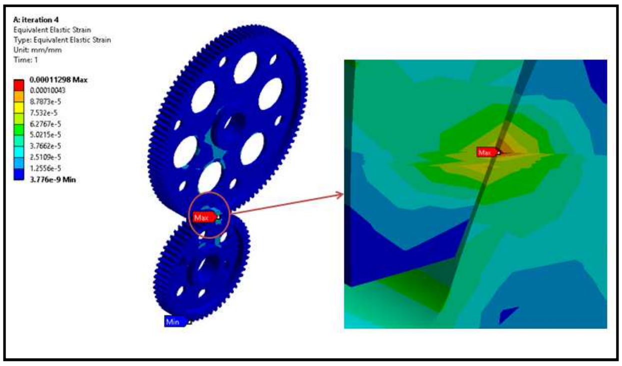

Figure 23: Strain Plot in Iteration 4 Analysis

Maximum strain of 0.000113 is observed in the FEA of gear design 4 which is final optimized design.

Table 8: Experimental and Testing Comparison

Parameter	Analysis Result	Experimental Result	% Error
Strain	0.000113	0.000104	7.96

6. CONCLUSIONS

It can be clearly observed that 30.01% of Equivalent stress, 10.21% of contact pressure is reduced& 12.51 % of total mass is reduced when compared to baseline gear assembly with modified design. Creating holes and reducing the thickness of the gear has huge impact on the weight of the gears while fillet radius has the large impact on the contact stresses of the gear. By changing the current conventional design parameters we can achieve better performance of the gear under contact pressure as well as in equivalent stress while reducing the weight of the gear. FEA results of contact pressure are in conformance with the theoretical calculations. Optimized model results are in conformance with the testing results. Strain values of the FEA and practical testing are matching with the error of 7.96 % which is acceptable as less than 10 % error.

7. ACKNOWLEDGMENT

I take this opportunity to thank Prof. S. D. Katekar and Prof. P. P. Kulkarni for valuable guidance and for providing all the necessary facilities, which were indispensable in completion of this work. Also I sincerely thanks to all the authors who worked on chain link weight optimization and material optimization.

REFERENCES

1. *“Study of Stress Relief Features at Root of Teeth of Spur Gear”, Dhavale A. S., Abhay Utpat, IJERA ISSN: 2248-9622 Vol. 3, Issue 3, May-Jun 2013, pp.895-899.*
2. *“Analysis of stress due to contact between spur gears”, rubén d. Chacón, advances in computational intelligence, man-machine systems and cybernetics, ISBN: 978-960-474-257-8.*
3. *“Design and finite element method analysis of gear mechanism for raw material crushing in plastic industry”, prof. Ajitabhpateriya, IJPRET, 2015; volume 3 (7): 48-55.*
4. *“Contact Stress Analysis of Spur Gear Teeth Pair”, Ali Raad Hassan, World Academy of Science, Engineering and Technology, Vol: 3, No:10, 2009.*

5. *"Modelling and Finite Element Analysis of Spur Gear", VivekKaraveer Ȧ, International Journal of Current Engineering and Technology ISSN 2277 – 4106.*

6. *"A Study on the Influence of using Stress Relieving Feature on Reducing the Root Fillet Stress in Spur Gear", Nidal H., Proceedings of the 2014 International Conference on Mathematical Methods, Mathematical Models and Simulation in Science and Engineering.*

7. *"Contact stress analysis for a pair of mating gears", Seok-Chul Hwang, Mathematical and Computer Modelling 57 (2013) 40–49.*

8. *"A Finite Element Approach to Bending, Contact and Fatigue Stress Distribution in Helical Gear Systems". S. Jyothirmai, Procedia Materials Science 6 (2014) 907 – 918.*

9. *"An investigation of contact stresses and crack initiation in spur gears based on finite element dynamics analysis", W. J. Qin, International Journal of Mechanical Sciences 83 (2014)96–103.*

10. *"Dynamic analysis of spur gear teeth", V. Ramamurti, compvrers & sfrucrures vol. 29, no. 5, pp. 831-843. 1988.*

11. *"A finite element analysis of bending stresses induced in external and internal involute spur gears", j. D. Andrews, journal of strain analysis vol. 26 no 3 1991.*

12. *"Comparative Finite Element Analysis of Reconstructed New and Worn Tooth of Spur gear", Atul Kumar, P. K. jain and P. M. Patha., Proceedings of the 1st International and 16th National Conference on Machines and Mechanisms (iNaCoMM2013), IIT Roorkee, India, Dec 18-20 2013.*

13. *"Analysis of Spur Gear Using Finite Element Analysis", Mrs. C.M. Meenakshi, akash Kumar, ApoorvaPriyadardsi, Digant Kumar Dash and Hare Krishna, Middle-East Journal of Scientific Research 12 (12): 1672-1674, 2012 ISSN 1990-9233.*

14. *''Contact stress analysis of spur gear '', Mr. Bharat Gupta, Mr. Abhishek choubey, 2012, International Journal of Engineering Research & Technology (IJERT), ISSN: 2278-0181, Vol. 1 Issue 4, June – 2012.*

15. *"Modal and stress analysis of gear train design in portal axle using finite element modeling and simulation", Jong Boon Ooi1,Xin Wang, Ching Seong Tan, Jee-Hou Ho, Ying Pio Lim, Journal of Mechanical Science and Technology 26 (2) (2012) 575~589.*

16. *"Contact Stress Analysis of Helical Gear by Using AGMA and ANSYS", S. Sai Anusha, P. Satish Reddy, P. Bhaskar, M Manoj, International Journal of Science Engineering and Advance Technology, Vol.2, Issue 12, (2014).*

17. *www-gateway.vpr.drexel.edu/files/bioengg_instrumentation/site/ElectricalResistance StrainGauge.html*

18. *Sendilvelan, S., & Prabhahar, M. Pre-Stress Modal Analysis Of A Centrifugal Pump Impeller For Different Blade Thicknesses.*